CENTRAL SCIENTIFIC COMPANY

OPERATING INSTRUCTIONS

Kundt's Apparatus
No. 84865

1. Introduction

The Kundt's apparatus is a device used to measure the velocity of sound transmission in metal rods, and in gases such as carbon dioxide and air. This is accomplished by measuring the wave length of the sound which a vibrating rod produces in air.

2. Theory

Sound is propagated by longitudinal waves traveling in a material medium. The velocity **v** of any wave is given by

$$v = n\lambda \qquad (1)$$

where **n** is the frequency and λ the wave length of the wave. In the case of longitudinal waves, the wave length is the distance between two consecutive compressions or rarefactions while the frequency is the number of compressions or rarefactions that pass any point in the medium per second.

An example of a longitudinal wave is shown in Figure 1(a), where the open circles represent a series of equidistant particles and the closed circles represent the displaced positions of these particles at some instant of time.

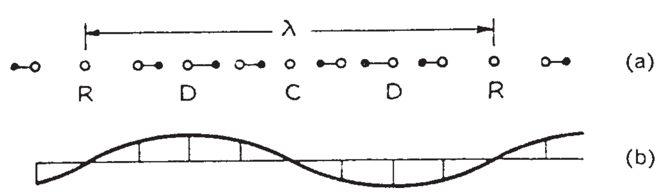

*Figure 1 (a) Equilibrium and displaced positions of particles in a longitudinal wave showing compressions **C**, rarefactions **R**, and places of normal density **D**. (b) Curve in which the ordinates are equal to the corresponding horizontal displacements in (a).*

The letters **R**, **C**, and **D** show the positions of the rarefactions, compressions, and places of normal density in the wave. A more convenient model, Figure 1(b), of the wave is shown by drawing a curve where the ordinates are equal to the corresponding horizontal displacements of Figure 1(a). In this representation a rarefaction occurs at a place having maximum positive slope, a condensation at a place of maximum negative slope, and a place of normal linear density where the slope of the curve is zero.

Equation (1) is a kinematical expression for the velocity of sound, and leaves much to be desired. It does not suggest whether or not sounds of different frequency have the same velocity. Nor does it give any information concerning whether the velocity of sound from a moving source is equal to, or different from, the velocity of a similar sound originating at a stationary source. Theoretical answers to these questions can be obtained only from dynamical considerations. The velocity of sound depends solely upon the properties of the material in which it is propagated; it is independent of frequency and the condition of motion of the source.

Since sound waves consist of a succession of compressions and rarefactions, the velocity of only one of them needs to be calculated to derive an algebraic relation for the velocity of sound. This can be done by equating the momentum change of a mass of gas to the impulse that produces it. First the bulk modulus **E** of a gas needs to be defined. By definition it is

$$E = -\Delta P / (\Delta V / V) = -V (\Delta P / \Delta V) \qquad (2)$$

where **ΔP** is the change in pressure of the gas, **V** is its volume, and **ΔV** is the change in volume corresponding to **ΔP**. The negative sign indicates that an increase in pressure produces a decrease in volume.

Let **S** be the cross-sectional area of the tube and **Δx** represent the movement of a disk that produces a compression in the gas. Then **ΔV = SΔx**. Likewise, if **Δt'** is the time required by the disk to produce the compression **ΔV**, and if **v** is the velocity with which this compression is increased, the volume affected by this compression in the time **Δt'** is

$$V = S (v\Delta t') \qquad (3)$$

The values in Equation (2) give the increase in pressure of gas as

$$\Delta P = -E (\Delta V / V) = -E\Delta x / (v\Delta t') \qquad (4)$$

The force ***f*** exerted by the disk in producing this increase in pressure is

$$f = S\Delta P = -SE\Delta x / (v\Delta t') \qquad (5)$$

and the impulse imparted to the gas by this force is

$$f\Delta t' = -SE\Delta x / v \qquad (6)$$

Let the momentum acquired by the center of mass of the gas be Δ(mu). Then

$$\Delta(mu) = \rho V (\Delta x / \Delta t') = \rho S v \Delta x \qquad (7)$$

where ρ is the density of gas. According to Newton's second law, the algebraic sum of the impulse and the change of momentum vanishes. Thus

$$-SE\Delta x / v + \rho S v \Delta x = 0 \qquad (8)$$

or

$$v = \quad E / \rho \qquad (9)$$

Experiments show that the elastic modulus that gives the correct value of the velocity from this formula is the adiabatic elastic modulus. The propagation of sound is an adiabatic process because the changes in pressure from compression to rarefaction occur too rapidly for transfers of energy to take place. The equation for an adiabatic curve is

$$PV^{\gamma} = C \qquad (10)$$

where **C** is a constant and γ is the ratio of the specific heat of the gas at constant pressure to its specific heat at constant volume. Thus

$$(P + \Delta P)\,(V - \Delta V)^{\gamma} = PV^{\gamma} \qquad (11)$$

The second factor of the left-hand side of this equation can be expanded by the binomial theorem. Because of the smallness of **ΔV**, it is accurate to keep only two terms of this expansion. Then Equation (11) yields the relation

$$-V\,(\Delta P / \Delta V) = \gamma P \qquad (12)$$

Thus

$$E = \gamma P \qquad (13)$$

and

$$v = \gamma P / \rho \qquad (14)$$

If suitable values for air under normal conditions of temperature and pressure are taken to be γ **= 1.40**, and ρ **= 0.001293 gm/cm³**, the velocity $\mathbf{v_o}$ of sound in air is given by this formula as 331.20 meters per second. The experimental value of 0°C is 331.70 meters per second.

By Boyle's law the quantity **P /** ρ is constant for a gas kept at constant temperature so the velocity of sound in a gas is independent of the pressure when the temperature is constant. The effect of temperature on the velocity of sound may be derived from a consideration of the general gas law. If $\mathbf{P_o}$ and ρ_o are the pressure and density of a gas at 0°C and **P** and ρ the pressure and density at some temperature t°C, then

$$P_o\,(1 + at) / \rho_o = P / \rho \qquad (15)$$

where **a** is the coefficient of expansion of an ideal gas at constant pressure and is approximately equal to 0.00367 per degree centigrade. If $\mathbf{v_o}$ is the velocity of sound in a gas at 0°C and $\mathbf{v_t}$ the velocity at temperature t°C, then combining Equations (14) and (15) yields

$$v_t = v_o \quad 1 + at \qquad (16)$$

Since **at** is a small quantity compared to unity, the first two terms in the binomial expansion of the square root are a sufficiently accurate approximation, so

$$v_t = v_o\,(1 + 0.00183\,t) \qquad (17)$$

When sound waves enter a column of air confined in a tube, reflection of the waves takes place at the end of the tube. The air in the tube is then acted upon by two similar sets of waves traveling in opposite directions. If the air column is of suitable length, the two oppositely traveling waves produce standing waves. Figure 2 shows a way of producing standing waves in the air confined in a tube closed at the end **E**.

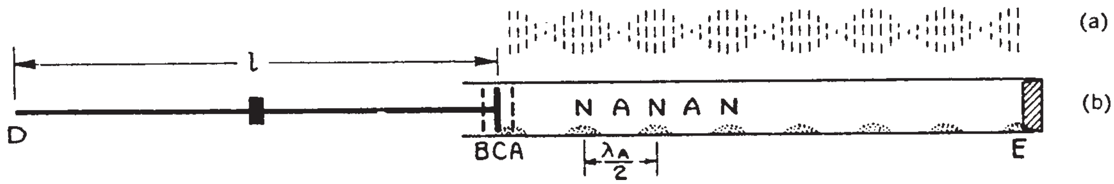

*Figure 2 (a) Appearance of dust heaps when standing waves are produced in air column. (b) Schematic diagram of apparatus, showing nodes **N** and antinodes **A**.*

Longitudinal waves are produced in the air column by means of a disk **C** attached to a uniform rod **CD** that can be set into longitudinal vibration. The amplitude of the vibrating disk moves from **B** to **A**, a compression of the air is sent down the tube with the velocity of sound. This compression is reflected from the closed end **E** and travels to **C**, where it is reflected again. If it arrives at **C** at a time when the disk is in a position to produce a compression, then resonance takes place between the vibrating disk and the air column. The succession of compressions and rarefactions sent out by the vibrating disk then produce standing waves in the air column. The production of standing waves by two oppositely traveling waves is shown in Figure 3.

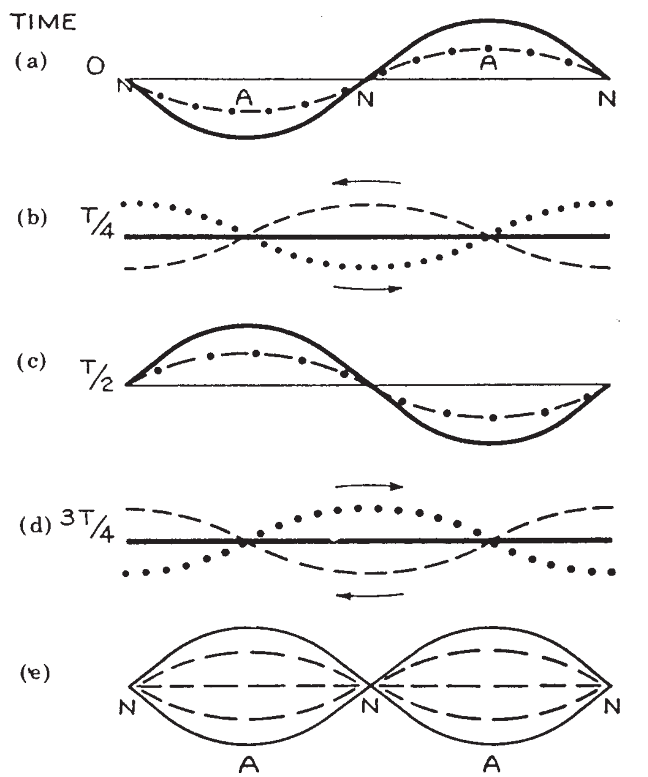

*Figure 3 Production of standing waves by two progressive waves traveling to the left and right respectively. Durves (a), (b), (c), (d) show the displacements of the particles due to the two waves and the resultant displacement at fractions of a period **O**, **T / 4**, **T / 2** and **3T / 4**. Curve (e) shows the position of the nodes **N** and the antinodes **A** in the stationary waves.*

In this diagram the dashed and dotted curves represent waves traveling to the left and right. Since the resultant displacement of the air particles at each point is the sum of the displacements due to the individual waves, and since the two corresponding displacements are equal everywhere, the resultant displacement curve, shown by the thick line in Figure 3(a), is similar to the individual curves but of double amplitude. A quarter of a period later, Figure 3(b), each wave will have progressed a quarter of a wave length so the displacements at each point are equal and opposite and the resultant displacement is zero everywhere. Figures 3(c) and 3(d) show the individual and resultant displacements of the air particles at one half and three quarters of a period. The resultant displacements for a whole period are shown in Figure 3(e), where at certain points such as **N**, called nodes, there is no displacement of the air particles while at others such as **A**, called antinodes, there is a maximum displacement. The nodes are places of maximum pressure changes — that is, the places where the compressions and rarefactions in the standing waves occur. The antinodes are places where the pressure or the density of the air remains constant.

Standing waves are also set up in the vibrating rod. The center of the rod, which is rigidly clamped, is a node of displacement while the ends of the rod, which are free to vibrate, are antinodes of displacement. If the only nodal point on the rod is at the center, then the wave length of the standing waves is twice the length of the rod and the rod is vibrating with its fundamental frequency. This fundamental frequency **n** is given from Equation (1)

$$n = v_R / \lambda_R = v_R / 2l \qquad (18)$$

where $\mathbf{v_R}$ is the velocity, $\lambda_\mathbf{R}$ the wave length of the longitudinal waves in the rod, and **l** the length of the rod. If the vibrations from the bar are transmitted to the air in a glass tube closed at one end, the frequency of the waves set up in the air is the same as that in the rod. If $\mathbf{v_t}$ is the velocity of sound in air at some temperature t°C and $\lambda_\mathbf{A}$ the wave length of the standing waves in the air column, then from Equation (1)

$$v_t = nl_A \qquad (19)$$

These standing waves can be detected by placing cork dust in the tube. This eventually collects at or around the nodes where the motion of the air is a minimum. If the wave length $\lambda_\mathbf{A}$ is measured and $\mathbf{v_t}$ is known from Equation (17), the frequency **n** can be calculated from Equation (19).

The accurate measurement of the wave length of these standing waves is difficult because the exact center of the dust heaps cannot be determined definitely. A fairly good result can be obtained by taking the average measured over a number of dust heaps. The best representative value for the constant interval, the distance between the dust heaps, can be obtained by using the equal interval method. To apply this method in the experiment, lay a meter stick alongside the glass tube and read off the positions in the centers of the dust heaps. The difference between any two successive readings should be constant, but owning to experimental errors is not exactly so. Suppose twelve readings are taken. Then take the difference between reading seven and reading one ($\mathbf{R_7 - R_1}$), reading eight and reading two ($\mathbf{R_8 - R_2}$), and so on up to that between reading twelve and reading six ($\mathbf{R_{12} - R_6}$). The average of these differences gives the average distance between six dust heaps. Dividing this latter value by six gives the best value for the distance between the dust heaps, i.e., half a wave length.

3. Description

The apparatus consists of a glass tube, clamped by coiled springs in grooves on a sheet metal support, and an aluminum rod firmly clamped at its center. The glass tube is 100cm in length and 3cm in diameter; the aluminum rod is 90cm in length. The density of the aluminum rod is

2.7 g/cm^3. The unique design of the rod's clamping device allows the user to raise or lower the rod, to turn it for centering or to release it for moving back and forth until the proper operating position is found. Once clamped, the rod is held firmly in place.

A disk, slightly smaller in diameter than the glass tube, is attached to the end of the aluminum rod in the tube. A stopper closes the other end of the glass tube.

4. Operation

The apparatus can be adjusted for measuring a wide range of wave lengths by sliding the tube in its spring holders. Fine cork dust or dried lycopodium powder may be used in the glass tube to show the patterns formed by nodes and antinodes.

Place a thin layer of very dry cork dust or lycopodium powder at least two millimeters wide as uniformly as possible inside the tube. Put the tube in position with the stopper inserted.

The rod is clamped at its center and the disk on the end of the rod is placed a few centimeters inside the tube but not touching the sides of the tube. The tube must be dry so the cork dust or lycopodium powder will not stick to the sides. If this should occur, dry the tube and the cork dust or powder. Place a small amount of rosin on the leather. Then, gripping the rod with rosined leather, pull towards the end of the rod. This sets the rod into longitudinal vibration and a note of high pitch will be heard. If the dust does not collect into heaps, slide the glass tube a short distance and again set the rod into vibration. With a little adjustment the dust heaps can be obtained. If the tube is gently rotated a small amount on its axis so the dust is raised on the side of the tube, the dust heaps are more readily formed. Using the meter stick as described above, record the position of the center of each dust heap.

Replace the air in the tube with carbon dioxide by allowing a stream of this gas to pass through the tube for two or three minutes. Repeat the experiment as before, recording the position of the centers of each dust heap. Note the temperature at which the experiment is performed. Measure the length of the rod.

5. Calculations

From the measurements of the distances between the dust heaps, determine the wave lengths of the standing waves produced in air and carbon dioxide by the method of equal intervals. Using Equation (17) and the velocity of sound at 0°C, calculate the velocity of sound in air at the temperature at which the experiment is performed. From Equation (1) calculate the frequency of the waves and, using this value, find the velocity of the longitudinal waves in the rod using the necessary data and Equation (18).

Optional

1. From tables find Young's modulus and the density of the rod. Use these values to calculate the velocity of sound in the rod. Compare with the experimental result.

2. Calculate γ for air and for carbon dioxide, using Equation (14).

6. Questions

1. Explain why the modulus of elasticity of a gas used in Equation (14) is γ **P** and not **P**.

2. If the rod had been twisted so as to set it into torsional vibration, would the frequency be the same as when the rod was set into longitudinal vibration? Why?

3. If the end of the tube had been left open instead of being closed, would standing waves have been set up and, if so, would the internodal distances have been changed? Explain.

4. Derive the equation **v = n**λ for the velocity of a wave.

5. Show that both sides of Equation (1) have the same dimensions.

6. Calculate the velocity of sound in water taking the necessary data from tables.

7. What property of rosin makes it good to use in this experiment?

8. The disk is an antinode of motion for the vibrating rod but a node for the vibrating air column. Explain.

9. (a) Show that the work done in compressing the gas in a Kundt's tube when the disk on the end of the rod moves through a distance Δx is $\mathbf{SE\ (\Delta x)^2 / 2\ l}$ where **l** is the length of the tube.

   (b) Show that the gain in kinetic energy of gas is

   $\rho S\ l / 2\ (\Delta x / \Delta t')^2$

   (c) Show by equating the work done to the gain in kinetic energy that the velocity of sound is given by Equation (9).

7. Maintenance

The Kundt's Apparatus needs no special maintenance. If you should experience any difficulty with this apparatus, please contact Central Scientific Company, giving details of the problem. To ensure better service, please do not return any item to Central Scientific Company until we have sent you authorization.

8. Accessories

Description	Cat. No.
Lycopodium Powder	84855-02
Cork Dust	84875

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